Higher Algebra I — Fall 2005

Gerald Hoehn

Problem sheet 5 September 23, 2005

Problem 1: Prove *Cauchy's Theorem:* Let p be a prime divisor of the order of a finite group G. Then there exists an element $a \in G$ of order p.

Problem 2: Prove that there is no simple group of order pqr, where p, q and r are distinct primes.

Problem 3: Prove that there is no simple group of order 2907.

Problem 4: Let A be a finite abelian group and let p be a prime. The p^{th}-power map is the homomorphism defined by

$$\varphi : A \longrightarrow A, \qquad a \mapsto \varphi(a) = a^p.$$

Let A^p be the image and let A_p be the kernel of φ, respectively. Prove that $A/A^p \cong A_p$. (Hint: Show that both groups are elementary abelian p-groups of the same order. An elementary abelian p-group is an abelian group where each element besides the identity has order p.)

Problem 5*: Prove that there is no simple group of order 1004913. (Hint: Find all possible values for the numbers n_p of p-Sylow subgroups and exclude them one by one. Finally, use a permutation representation of degree 819 to exclude the last case.)