

Differential Topology — Spring 2014

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Problem 1: Let M^n be an n -dimensional compact manifold, $f : M \rightarrow \mathbf{R}^{n+1}$ be differentiable, and $0 \notin f(M)$. Show that there exists a line through the origin of \mathbf{R}^{n+1} which only meets finitely many points of $f(M)$.

Problem 2: Let $f : M \rightarrow \mathbf{R}^p$ be a differentiable map and $N \subset \mathbf{R}^p$ be a differentiable submanifold. Show that for each $\epsilon > 0$ there exists a $v \in \mathbf{R}^p$ with $|v| < \epsilon$, so that the map $M \rightarrow \mathbf{R}^p$, $x \mapsto f(x) + v$, is transverse to N .

Hint: Consider the map $M \times N \rightarrow \mathbf{R}^p$, $(x, y) \mapsto x - f(y)$.

Problem 3: Let $M^k \subset \mathbf{R}^{n+1}$ be a compact submanifold and $n \geq 2k$. Show that, for the projection $\pi : \mathbf{R}^{n+1} \rightarrow H^n$ onto a suitable hyperplane of \mathbf{R}^{n+1} , the restriction $\pi|_M : M \rightarrow H$ is an immersion.

Hint: Consider the $(2k - 1)$ -dimensional manifold PTM , whose elements are the 1-dimensional subspaces of the tangent spaces of M , and study the canonical map $\alpha : PTM \rightarrow \mathbf{RP}^n$ (the n -dimensional real projective space).

Problem 4: Show that the ring $\mathcal{E}_0(\mathbf{R}^n)$ of germs of differentiable functions on \mathbf{R}^n at 0 has zero divisors.