Differential Topology — Spring 2014

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Problem 1: (a) Show that for each natural number $n \geq 0$ there is a flow on S^1 with exactly n fixed points.

(b) Show that on the torus $S^1 \times S^1$ there exists a vector field for which no orbit of the associated flow is a submanifold of $S^1 \times S^1$. *Hint:* $S^1 \times S^1 = (\mathbf{R} \times \mathbf{R})/(\mathbf{Z} \times \mathbf{Z})$. Consider a specific constant vector field on $\mathbf{R}^2$.

Problem 2: Show that every submanifold diffeomorphic to S^1 inside a differentiable manifold M arises as the orbit of a global flow on M. *Hint:* Partitions of unity.

Problem 3: Show that a bounded vector field defined on $\mathbf{R}^n$ is globally integrable.

Problem 4: Let E and F be vector bundles over X. Prove that the kernel and image of a bundle isomorphism $f : E \longrightarrow F$ of constant rank are subbundles of E respectively F. *Hint:* One has to show that for each point $p \in X$ there exists bundle charts (ϕ, U) for E and (ψ, U) for F such that for every $x \in U$ the map $\psi \circ f \circ \phi^{-1} : U \times \mathbf{R}^m \longrightarrow U \times \mathbf{R}^n$ is given by $(u, (v_1, \ldots, v_m)) \mapsto (u, (v_1, \ldots, v_k, 0, \ldots, 0))$. Use a method similar as was done for the local form of differential maps.